Sugar Factory Control Strategy

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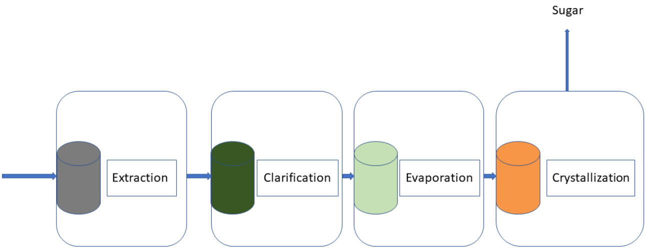
Mathematics in Industry Study Group 2023

January 20, 2023

Outline

Problem Statement

- In a sugar factory, there are a series of unit operations, and the process needs to be controlled so that the buffers neither run empty nor overflow.



Problem Statement

- The constituents of the cane include fibre, brix and water, whose exact amounts depend on the time of the year, the variety of cane and the growing conditions.

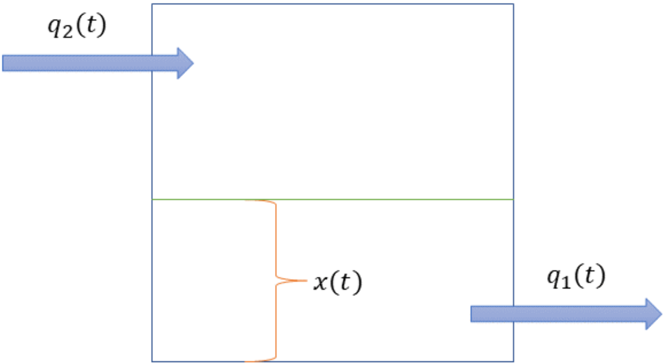
- Minimum throughput is required to maintain performance in some units.

- The process is limited by some bottlenecks in the different units, which need to be controlled for optimal sugar production.

Objective

- To investigate whether or not the buffer tank theory can be applied to the more complex multi-component streams in sugar factories.
- To minimise the size of the buffer tanks required to accommodate the normal fluctuations in operation.
- To find out whether increasing the size of buffer tanks is an economic alternative.

Single compartment model



We define $x(t)$ as follows

$$x(t) = \int_0^T [q_2(t) - q_1(t)] \; dt \tag{1}$$

which follows that

$$\dot{x}(t) = q_2(t) - q_1(t)$$

with $l \leq x(t) \leq k$.
We also have that

$$q_1 \leq U_{q_1}$$

.
The objective is to maximize the throughput,

$$\max \int_0^T q_1(t) \; dt \tag{2}$$

Optimal control model for a single compartment.

$$max \int_0^T \left[q_1(t) + (u_{q_1} - q_1)^2 + (x(t) - l)^2 + (k - x(t))^2\right] dt$$

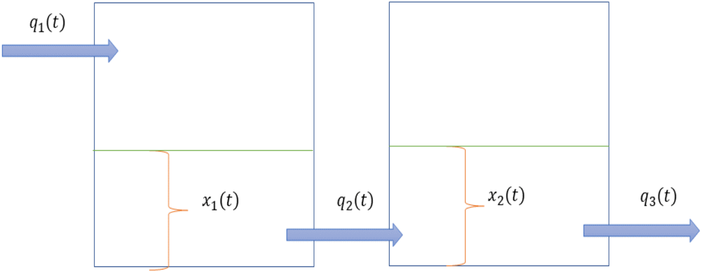
subject to:

$$\dot{x}(t) = q_2(t) - q_1(t), \qquad x(0) = x_0$$

and

Two compartmental model

We model the process as a coupled tank system with three valves



Here x_1 and x_2 are the fluid level in tanks 1 and 2, respectively, and q_1, q_2 and q_3 are the flow rate.

The flow balance equation gives the following differential equations for each tank:

$$\frac{dx_1}{dt} = q_1 - q_2,$$
$$\frac{dx_2}{dt} = q_2 - q_3.$$

Assuming we are dealing with steady, non-viscous, imcompressible fluid - the outlet flow is proportional to the square root of the tank level

$q_2 = a_1\sqrt{x_1 - x_2}$ and $q_3 = f(x_2, t) = \frac{a_2}{2}\sqrt{x_2}(sin(\alpha t) + 1)$,
where a_1 and a_2 are proportionality constants.

Hence, the flow balance can be rewritten as the nonlinear dynamical system

$$\frac{dx_1}{dt} = u - a_1\sqrt{x_1 - x_2} + \omega_1, \tag{3}$$

$$\frac{dx_2}{dt} = a_1\sqrt{x_1 - x_2} - \frac{a_2}{2}\sqrt{x_2}(sin(\alpha t) + 1) + \omega_2. \tag{4}$$

Here, we introduce the random variables, ω_1, and ω_2, to model the random disturbance of the system.

The problem of controlling fluids in tank 2 can be formulated as the optimal control problem.

- Find the optimal control input, u, which is the flow rate, to minimize the cost function

$$J = \frac{1}{2}\int_0^T \left[(x_2(t) - x_2^s)^2 + (u(t) - u^s)^2\right] dt, \tag{5}$$

where x_2^s and u^s are the steady state values.

A simple optimal control model for sugar processing

$$\min \left\{ J = \frac{1}{2} \int_0^T \left[(x_2(t) - x_2^s)^2 + (u(t) - u^s)^2 \right] dt \right\},$$

subject to:

$$\frac{dx_1}{dt} = u - a_1\sqrt{x_1 - x_2} + \omega_1,$$
$$\frac{dx_2}{dt} = a_1\sqrt{x_1 - x_2} - \frac{a_2}{2}\sqrt{x_2}(sin(\alpha t) + 1) + \omega_2.$$

and the initial condition $(x_1(0), x_2(0)) = \left(x_1^0, x_2^0\right)$.

We solve the system using the Maximum Pontryagan's Principle.

- For a given system and the optimal criterion, let u^* be an optimal control, then there exists a costate variable, λ, which together with the state variable, x satisfies the canonical Hamiltonian equations,

$$\dot{\mathrm{x}} = \nabla_\lambda H(t, \mathrm{x}, \mathrm{u}, \lambda), \qquad \mathrm{x}(0) = \mathrm{x}_0,$$
$$\dot{\lambda} = \nabla_{\mathrm{x}} H(t, \mathrm{x}, \mathrm{u}, \lambda), \qquad \lambda(T) = 0,$$

where

$$H(t, \mathrm{x}, \mathrm{u}, \lambda) = L(t, \mathrm{x}, \mathrm{u}) + \lambda^T F(t, \mathrm{x}, \mathrm{u})$$

and

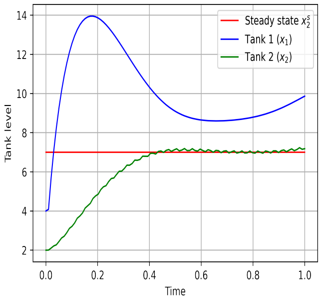
$$H(t, \mathrm{x}^*, \mathrm{u}, \lambda^*) \leq H(t, \mathrm{x}^*, \mathrm{u}^*, \lambda^*).$$

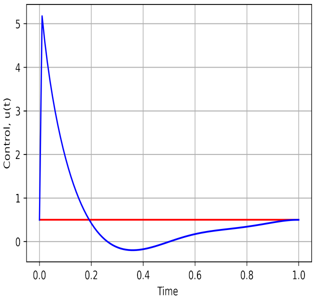
The algorithm

1. Initialize the control variable, u^0.
2. Solve the state equation using the forward substitution method
3. Solve costate equation using the Backward substitution method
4. Then update
$$\mathrm{u}^k = \mathrm{u}^{k-1} + \alpha \frac{\partial H}{\partial \mathrm{u}}$$
5. Repeat steps 2-4 until Optimality is attained, $\left\|\frac{\partial H}{\partial \mathrm{u}}\right\| \leq \epsilon$

Two compartmental model

Some results





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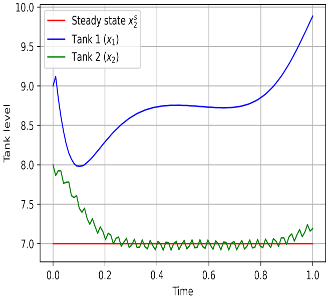

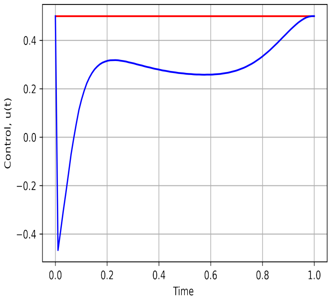

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  - **A:** Disturbance attenuation - tanks installed between units to avoid propagation of disturbances for continuous processes.
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- In category **A**, there are 2 fundamental different disturbances, namely, in **quality** and **flow rate**.

- Two approaches to dampen category **A** disturbances:
  1. Quality disturbances e.g. in concentration/temperature, dampen by **mixing**. Such buffer tanks are called mixing tanks.
  2. Flow rate disturbances e.g. in the feed rate, dampen by **changing the volume (level variation)**. Such buffer tanks are called surge tanks, hold up tanks, intermediate storage vessels etc.

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- In both cases, the tank volume is exploited, and a larger volume gives a better dampening.
    a. In the first case, mixing of larger volume means that the inflow entering during a longer period is mixed altogether.
    b. In the second case, larger level variations are allowed.

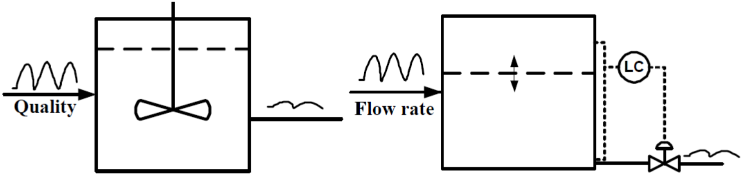
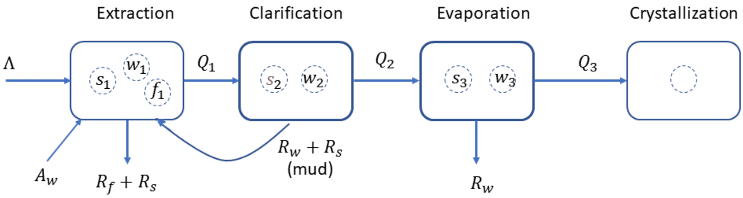


Figure: 1. Averaging by mixing (mixing tank) 2. Averaging level control (surge tank)

- In the design of buffer tanks, the **residence time or hold up** time is used as a measure instead of volume.

$$\tau = \frac{\text{amount (or concentration) of material in reservoir}}{\text{flow rate}} \tag{6}$$

Sugar cane processing



- We want to answer the following questions: *(i.)* when should a buffer be installed to avoid propagation of disturbances and *(ii.)* how larger should the tank be?

- The buffer tank (with transfer function $h(s)$) should modify the disturbance, d, such that the modified disturbance

$$d'(s) = h(s)d(s) \tag{7}$$

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- The buffer tank design problem can be solved in two steps:
  1. **Step 1:** Find the required transfer function $h(s)$. Typically,

$$h(s) = \frac{1}{(\tau s + 1)^n} \tag{8}$$

  2. **Step 2:** Find a physical realization of $h(s)$ (tank volume and possibly level control tuning).

Model of sucrose concentrations in the sugar production process:

$$\frac{dS_1}{dt} = \frac{1}{V_1}\left(\Lambda_s - Q_1 S_1 - R_s^1 S_1 + R_s^2 S_2\right) \tag{9}$$

$$\frac{dS_2}{dt} = \frac{1}{V_2}\left(Q_1 S_1 - Q_2 S_2 - R_s^2 S_2\right) \tag{10}$$

$$\frac{dS_3}{dt} = \frac{1}{V_3}\left(Q_2 S_2 - Q_3 S_3\right) \tag{11}$$

subject to $\quad S_1(0) \geq 0,\ S_2(0) \geq 0$ and $S_3(0) \geq 0.$

- For identification of parameters, we solve the optimization problem as formulated below:
  Let $X = (S_1, S_2, S_3)^T$, and $f_1 = dS_1/dt$, $f_2 = dS_2/dt$, dS_3/dt in which the model can be written as

$$\dot{X} = F(X(t), \mu), \tag{12}$$

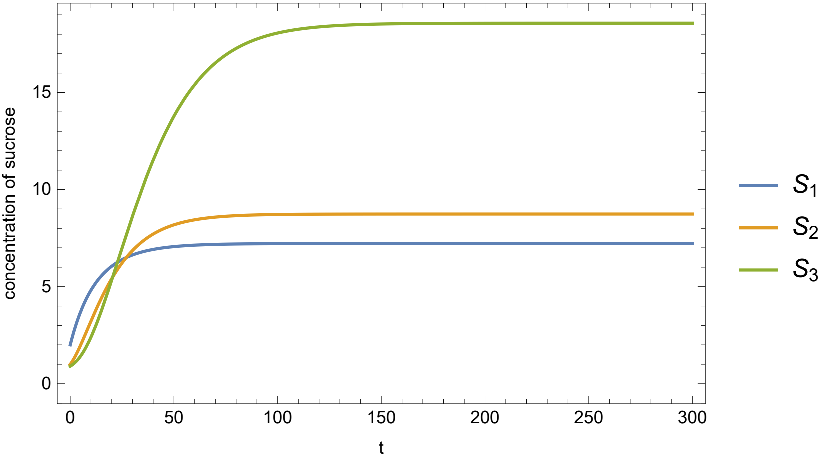
where $F = (f_1, f_2, f_3)'$ and μ is a column vector of parameters

$$\mu = (\Lambda_s, Q_1, ..., Q_3)$$

- The optimization problem is

$$\min_{\mu} \sum_{i=1}^{n} q(S^i(t) - S^{obs})^2 \tag{13}$$

"Some results"



Conclusion

- Consider multi-Compartmental model to include all the different stages in Sugar production process.
- Find the residence time for each processing stage.
- Relate the two approaches in determining whether increasing the size of buffer tanks is an economic viable option.

Thank You

for your attention.

Questions and comments are ...